RELATIVE EFFICIENCIES OF SOME ALTERNATIVE PROCEDURES IN TWO-STAGE SAMPLING ON SUCCESSIVE OCCASIONS

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Introduction

In repeat sample surveys, the techniques developed by Yates (1960), Patterson (1950), Eckler (1955) and others for single-stage sampling design were extended to two-stage sampling by Kathuria (1959), and Singh and Kathuria (1969). Singh (1968), for sampling on three occasions, obtained estimates of mean for the current occasion as well as an overall estimate of mean for the three occasions. Tikkiwal (1964) extended the covariance conditions developed by Patterson and Eckler to two-stage sampling design.

With a two-stage sampling design, partial replacement of units from the sample drawn on the previous occasion can be done in three ways :

(*i*) Retain only a fraction p of the primary sampling units (psu's) with their samples of second stage units (ssu's) and select a fraction q of psu's $(q+p=1)$ afresh,

(*ii*) retain all the psu's from the preceding occasion but from each psu retain only a fraction p of the ssu's within them and select a fraction q of ssu's afresh,

(*iii*) retain a fraction p of the psu's and from each such psu, retain only a fraction r of the ssu's and select a fraction s of the ssu's afresh $(r+s=1)$.

Sampling patterns (*i*) and (*ii*) have been discussed by Singh and Kathuria (1969) and estimates of mean and variances of the estimates were obtained. Abraham *et al* (1969) obtained estimates of mean on the second occasion and of the change and the variances of these estimates using sampling pattern (*iii*).

In the present investigation we shall examine the relative efficiencies of the estimates obtained under the three sampling patterns given above. We shall also examine the relative efficiencies of some alternative replacement procedures when sampling is carried on to three occasions.

2. Sampling patterns (i) and (ii)

Estimate of mean on the current occasions

2.1. Let there be a population π consisting of N psu's wherein each psu consists of M ssu's. Let $y_{kl}^{(i)}$ be the value of the l-th ssu in the k-th psu drawn on the i-th occasion ($i=1, 2, \ldots h$; $k=1, 2, \ldots N$; $l=1, 2, \ldots M$). We are required to estimate the population mean $\overline{Y}^{(h)}$, where

$$\overline{Y}^{(h)} = \frac{1}{NM} \sum_{k=1}^{N} \sum_{l=1}^{M} y_{kl}^{(h)}.$$

Define

$$S_b^{2(i)} = \frac{1}{(N-1)} \sum_{k=1}^{N} \left(\overline{Y}_{k.}^{(i)} - \overline{Y}_{..}^{(i)} \right)^2$$

as the mean square between the psu means in the population on the i-th occasion,

$$S_w^{2(i)} = \frac{1}{N(M-1)} \sum_{k=1}^{N} \sum_{l=1}^{M} \left(y_{kl}^{(i)} - \overline{Y}_{k.}^{(i)} \right)^2$$

as the mean square between ssu's within psu's in the population on the i-th occasion ($i=1,2,\ldots h$),

$$\rho_b^{(i,j)} S_b^{(i)} S_b^{(j)} = \frac{1}{(N-1)} \sum_{k=1}^{N} \left(\overline{Y}_{k.}^{(i)} - \overline{Y}_{..}^{(i)} \right) \left(\overline{Y}_{k.}^{(j)} - \overline{Y}_{..}^{(j)} \right)$$

as the covariance between psu means in the population on the i-th and j-th occasions and

$$\rho_w^{(i,j)} S_w^{(i)} S_w^{(j)} = \frac{1}{N(M-1)} \sum_{k=1}^{N} \sum_{l=1}^{M} \left(y_{kl}^{(i)} - \overline{Y}_{k.}^{(i)} \right) \left(y_{kl}^{(j)} - \overline{Y}_{k.}^{(j)} \right)$$

as the covariance between ssu's within psu's on the i-th and j-th occasions ($i \neq j=1,2,\ldots h$). For simplicity, we may assume N and M to be large such that terms of the order $\frac{1}{N}$ and $\frac{1}{M}$ are ignored. Further, we may assume that

$$S_b^{(i)} = S_b^{(j)} = S_b \text{ and } S_w^{(i)} = S_w^{(j)} = S_w.$$

We also assume that

$$\rho_b^{(i,j)} = \rho_b^{|i-j|} \text{ and } \rho_w^{(i,j)} = \rho_w^{|i-j|}$$

for all $i \neq j=1, 2, \cdot h$.

Also we write

$$S_b^2 + \frac{S_w^2}{m} = \alpha, \quad \rho_b S_b^2 + \rho_w \frac{S_w^2}{m} = \gamma.$$

Let the sample size on any occasion consist of n psu's each of them consisting of m ssu's. Let the sample size on the h-th occasion consist of np psu's with their samples of ssu's retained from the total sample drawn on the preceding occasion and nq psu's selected afresh from the population ($q+p=1$).

Let $\bar{y}_1^{(h-1)}$, $\bar{y}_1^{(h)}$ and $\bar{y}_2^{(h-1)}$, $\bar{y}_2^{(h)}$ denote the means on the $(h-1)$-*th* and *h-th* occasions based on *npm* and *nqm* units respectively. Using sampling pattern (*i*), Singh and Kathuria (1969) obtained the following estimate of $\bar{Y}^{(h)}$.

$$\bar{y}_{(h)} = c_h \left\{ \bar{y}_1^{(h)} + \frac{\Upsilon}{\alpha}\left(\bar{y}^{(h-1)}\right)\right\} + (1-c_h)\,\bar{y}_2^{(h)} \qquad \text{...(2.1.1)}$$

where

$$c_h = \frac{p}{\left[1 - \frac{\Upsilon^2}{\alpha^2}\,(q-p) - pc_{h-1}\,\frac{\Upsilon^2}{\alpha^2}\right]} \qquad \text{...(2.1.2)}$$

and $c_1 = p$. The variance of the estimate $\bar{y}_{(h)}$ is given by

$$V(\bar{y}_{(h)}) = (1-c_h)\,\frac{\alpha}{nq}. \qquad \text{...(2.1.3)}$$

Similarly, using sampling pattern (*ii*), the estimate for $\bar{y}^{(h)}$ is given by

$$E_{(h)} = g_h\left\{\bar{y}_1^{(h)} + \rho_w\left(\bar{y}^{(h-1)}\right) - \bar{y}_1^{(h-1)}\right\} + (1-g_h)\,\bar{y}_2^{(h)} \qquad \text{...(2.1.4)}$$

and its variance is given by

$$V(E_{(h)}) = \frac{S_b^2}{n} + (1-g_h)\,\frac{S_w^2}{nmq} \qquad \text{...(2.1.5)}$$

where

$$g_h = \frac{p}{[1-(q-p)\rho_w^2 - pg_{h-1}\rho_w^2]} \qquad \text{...(2.1.6)}$$

g_1 being equal to p.

Kathuria and Singh (1970) have examined the relative efficiency of the estimate obtained under sampling pattern (*ii*) in relation to the estimate obtained under sampling pattern (*i*) for $h=2$, 3, 4 and 5 for a set of values of

$$\rho_b,\ \rho_w,\ \phi\left(=\frac{S_w^2}{S_b^2}\right) \text{ and } q.$$

3. Sampling pattern (iii)

Estimate of mean on the second occasion

Suppose now that on the second occasion a fraction p of the n psu's and in each of np psu's, a fraction r of the ssu's were retained. The remaining units, *viz.*, nq psu's and ms ssu's in each of np psu's were replaced by the units selected afresh on the second occasion. Let

$\bar{y}_1'$ =mean per ssu on the first occasion based on *npmr* units which are common to two occasions,

$\bar{y}_2'$ =mean per ssu based on common units on the second occasion,

$\bar{y}_1''$ = mean per ssu on the first occasion based on *npms* units taken from common psu's on both occasions,

$\bar{y}_2''$ = mean per ssu based on these units on the second occasion,

$\bar{y}_1'''$ = mean per ssu on the first occasion based on *nqm* units which are in the sample on the first occasion only,

$\bar{y}_2'''$ = mean per ssu on the second occasion based on *nqm* units selected afresh on the second occasion.

Consider first the *np* common psu's. In each of these psu's, a fraction r of the ssu's were retained and a fraction s were selected afresh on the current occasion. Proceeding as in (2.1.4) to (2.1.6) in section 2.1 above, we may write an estimate of $\bar{y}_2$, denoted by $\bar{y}_{2c}$ and variance of the estimate based on common psu's by replacing p and q by r and s respectively. These are given by

$$\bar{y}_{2c}=g_2\{\bar{y}_2'+\rho_w(\bar{y}_{1c}-\bar{y}_1')\}+(1-g_2)\bar{y}_2'' \qquad \ldots(3.1)$$

and

$$V(\bar{y}_{2c})=\frac{S_b^2}{np}+(1-g_2)\frac{S_w^2}{npms} \qquad \ldots(3.2)$$

where $$g_2=\frac{r}{(1-\rho_w^2 s^2)}$$

and $\bar{y}_{1c}$ is the mean per *ssu* on the first occasion based on *np* psu's common to the two occasions.

Abraham *et al* (1969) obtained an unbiased estimate based on all the units on the second occasion given by

$$\bar{y}_w=f_2\bar{y}_{2c}+(1-f_2)\,\bar{y}_2'''+f_2'\left(\bar{y}_{1c}-\bar{y}_1'''\right) \qquad \ldots(3.3)$$

where f_2 and f_2' are constants chosen in a way such that $V(\bar{y}_w)$ is minimum and $\bar{y}_{2c}$ is as defined above. The values of f_2 and f_2' are given by

$$f_2=\frac{p\alpha^2}{\left[\alpha\left\{S_b^2+\left((1-g_2)+ps\right)\frac{S_w^2}{ms}\right\}-\beta^2q^2\right]}$$

$$f_2'=-\frac{q\beta}{\alpha}f_2,$$

where $$\beta=\rho_b S_b^2+g_2\,\rho_w\,\frac{S_w^2}{m},$$

α and γ are as already defined above.

The variance of the estimate $\bar{y}_w$ is given by

$$V(\bar{y}_w)=(1-f_2)\frac{\alpha}{nq}. \qquad \ldots(3\cdot4)$$

It would be interesting to compare the estimate $\bar{y}_w$ with the two estimates obtained under sampling patterns (*i*) and (*ii*) above. This would be done by working out the efficiency of $\bar{y}_w$ with respect to $\bar{y}_2$ and E_2 for different values of the parameters involved. Tables 1 and 2 give the relative efficiency of $\bar{y}_w$ in relation to the estimates $\bar{y}_2$ and E_2 respectively.

4. Sampling for three occasions—estimates of mean on the current occasion

When sampling is carried on to three occasions, then with the sampling pattern on the second occasion being as given in section 3 above, Kathuria (1970) considered three different sampling patterns for selection of *psu's* and *ssu's* on the third occasion. Briefly, the three sampling patterns are follows :—

Sampling Pattern I :

Time	1	XXX	XXXX			XXXXX		
Time	2	XXX		XXXX			XXXXX	
Time	3	XXX			XXXX			XXXXX
Sample fraction		*npmr*	*npms*	*npms*	*npms*	*nqm*	*nqm*	*nqm*

Sampling Pattern II :

Time	1	XXX	XXXX			XXXXX	
Time	2	XXX		XXXX			XXXXX
Time	3	XXX		XXXX			XXXXX
Sample fraction		*npmr*	*npms*	*npms*	*nqm*	*nqm*	*nqm*

Sampling Pattern III :

Time	1	XXX	XXXX		XXXX				
Time	2	XXX		XXXX		XX	XXX		
Time	3					XX		XXX	XXX
Sample fraction		*npmr*	*npms*	*npms*	*nqms*	*nqmr*	*nqms*	*nqms*	*nqm*

Under sampling pattern I, the same sample consisting of *npmr* units is retained for all the three occasions, the remaining units are selected afresh each time. Under sampling pattern II, while only *npmr* units are retained from first occasion to the second, all the *np psu's* with their sample of *ssu's* are retained from second occasion to the third. Under sampling pattern III, only *npmr* units are kept common between first two occasions and only *nqmr* units are kept common between the second and the third occasion.

Unbiased estimates of the population mean $\bar{Y}_3$ on the third occasion and their variances were obtained for each of the three sampling patterns. Without going into complicated algebra, we shall only write the expressions for various estimators and their variances under each of the three sampling patterns.

Sampling Pattern I :

Let $\bar{y}_1', \bar{y}_1'', \bar{y}_1'''$ and $\bar{y}_2', \bar{y}_2'', \bar{y}_2'''$ be the sample means as defined in section 3 above and let the corresponding sample estimates on the third occasion be denoted by $\bar{y}_3', \bar{y}_3'', \bar{y}_3'''$. We define by $\bar{y}_3^{(1)}$ as the unbiased estimate of $\bar{y}_3$ under sampling pattern I.

Estimate :

$$\bar{y}_3^{(1)}=f_3\bar{y}_{3c}^{(1)}+(1-f_3)\bar{y}_3'''+f_3'(\bar{y}_2-\bar{y}_2''') \qquad \text{...(4·1)}$$

where $\bar{y}_{3c}^{(1)}$ and its variance are obtained in a similar manner as given in (3·1) and (3·2) above and $g_3=\dfrac{r}{[1-(s-r)\,\rho_w^2-rg_2\rho_w^2]}$. The values of f_3 and f_3' obtained by minimising the variance of $\bar{y}_3^{(1)}$ are given by

$$f_3=\frac{p^2\alpha^2}{\left[p\alpha\left\{s_b^2+(ps+(1-g_3)q)\dfrac{s_w^2}{ms}\right\}-f_2q^2\left\{\left(\rho_bs_b^2+(1-g_2)g_3\rho_w\dfrac{s_w^2}{ms}\right)-\dfrac{q\beta}{\alpha}\left(\rho_b^2s_b^2+g_2g_3\,\rho_w^2\dfrac{s_w^2}{m}\right)\right\}^2\right]}$$

and
$$f_3'=-\frac{q\alpha}{p}f_3\left[\left(\rho_bs_b^2+(1-g_2)g_3\rho_w\frac{s_w^2}{ms}\right)-\frac{q\beta}{\alpha}\left(\rho_b^2s_b^2+g_2g_3\rho_w^2\frac{s_w^2}{m}\right)\right]$$

variance of $\bar{y}_3^{(1)}$:

$$V\left(\bar{y}_3^{(1)}\right)=(1-f_3)\,\frac{\alpha}{nq}. \qquad \text{...(4·2)}$$

Sampling Pattern II :

Estimate :

The estimate based on sampling pattern II may be written as

$$\bar{y}_3^{(2)}=k_3\,\bar{y}_{3c}^{(2)}+(1-k_3)\,\bar{y}_3'''+k_3'(\bar{y}_2-\bar{y}_2''') \qquad \text{.. (4·3)}$$

where $\bar{y}_{3c}^{(2)}$ is an estimate based on common *psu's* between the second and the third occasions and is given by

$$\bar{y}_3^{(2)}=h_3\bar{y}_3'+(1-h_3)\bar{y}_3''+j_3(\bar{y}_{2c}-\bar{y}_2''),$$

h_3 and j_3 obtained by minimising $V\left(\bar{y}_{3c}^{(2)}\right)$ are given by

$$h_3=\frac{r(1-\rho_w^2)}{(1-\rho_w^2+\rho_w^4 s^2)},$$

$$j_3=\frac{\rho_w^3 s^2}{(1-\rho_w^2+\rho_w^4 s^2)}$$

and $\bar{y}_{2c}$, $\bar{y}_2$ are as defined above.

The values of k_3 and k_3', obtained by minimising $V\left(\bar{y}_3^{(2)}\right)$ are given by

$$k_3=\frac{p^2\alpha^2}{\left[p\alpha\left\{s_b^2+\left(\frac{h_3q+pr}{r}+\frac{(1-g_2)j_3\rho_w q}{s}\right)\frac{s_w^2}{m}\right\}-f_2q^2\left\{\left(1-\frac{q\beta}{\alpha}\rho_b\right)\rho_b s_b^2+\left(\frac{1-g_2}{s}-\frac{q\beta}{\alpha}(h_3\rho_w+j_3g_2)\right)\rho_w\frac{s_w^2}{m}\right\}^2\right]}$$

and $$k_3'=\frac{-qk_3}{p\alpha}\left[\left(\rho_b-\frac{q\beta}{\alpha}\rho_b^2\right)s_b^2+\left\{\frac{1-g_2}{s}-\frac{q\beta}{\alpha}(h_3\rho_w+j_3g_2)\right\}\rho_w\frac{s_w^2}{m}\right]$$

Variance of $\bar{y}_3^{(2)}$:

$$V\left(\bar{y}_3^{(2)}\right)=(1-k_3)\frac{\alpha}{nq}. \qquad \ldots(4{\cdot}4)$$

Sampling Pattern III :

Under sampling pattern III, there are nq psu's common between the second and the third occasion. Further, within each of these nq psu's on the third occasion, mr ssu's are retained from the second occasion and ms ssu's are selected afresh $(r+s=1)$. A fresh sample of np psu's and within each of the np psu's a sub-sample of m ssu's is selected afresh without replacement from the population on the third occasion such that $p+q=1$.

Now let

$\bar{y}_2'^{(3)}$ = mean per ssu on the second occasion based on $nqmr$ units which are common with the third occasion,

$\bar{y}_3'^{(3)}$ = mean per ssu based on these units on the third occasion,

$\bar{y}_2''^{(3)}$ = mean per ssu on the second occasion based on $nqms$ units not common with third occasion,

$\bar{y}_3^{''(3)}$ = mean per ssu based on *nqms* units on the third occasion,

$\bar{y}_2^{'''(3)}$ = mean per ssu on the second occasion based on *npm* units,

$\bar{y}_3^{'''(3)}$ = mean per ssu on the third occasion based on *npm* units.

The sample mean $\bar{y}_2'''$ based on *nqm* units on the second occasion may also be written as $\bar{y}_2''' = r\bar{y}_2^{'(3)} + s\bar{y}_2^{''(3)}$.

Estimate :

Denoting by $\bar{y}_3^{(3)}$, the estimate based on sampling pattern III, we have

$$\bar{y}_3^{(3)} = l_3\bar{y}_{3c}^{(3)} + (1-l_3)\bar{y}_3^{'''(3)} + l_3'\left(\bar{y}_2''' - \bar{y}_2^{'''(3)}\right) \qquad ..(4{\cdot}5)$$

where $\bar{y}_{3c}^{(3)}$ is the estimate based on *nq* psu's common between the second and the third occasion. The value of l_3 and l', obtained by minimising $V\left(\bar{y}_3^{(3)}\right)$, are given by

$$l_3 = \frac{q\alpha^2}{\left[\alpha\left\{s_b^2 + (qs + (1-g_2)p)\frac{s_w^2}{ms}\right\} - p^2\left(\rho_b s_b^2 + \rho_w g_2\frac{s_w^2}{m}\right)^2\right]}$$

$$l_3' = \frac{-pl_3\left(\rho_b s_b^2 + \rho_w g_2\frac{s_w^2}{m}\right)}{\left(s_b^2 + \frac{s_w^2}{m}\right)}$$

Variance of $\bar{y}_3^{(3)}$:

$$V\left(\bar{y}_3^{(3)}\right) = (1-l_3)\frac{\alpha}{np}. \qquad ..(4{\cdot}6)$$

5. Relative efficiencies of different sampling patterns

It would be interesting to examine the efficiencies of the sampling pattern II and III in relation to sampling pattern I. Table 3 gives the relative efficiency of the estimate $\bar{y}_3^{(2)}$ w.r.t. the estimates $\bar{y}_3^{(1)}$ and Table 4 gives relative efficiency of the estimate $\bar{y}_3^{(3)}$ w.r.t. the estimate $\bar{y}_3^{(1)}$.

Under sampling pattern I, only *npmr* are common to all the three occasions, while under sampling pattern II, besides *npmr* units being common between first two occasions, there are *npm* units common between second and third occasions. The two sampling patterns were found to be equally efficient. Under sampling pattern III, there are no common units between first and the third occasion while *nqmr* units are common between second and the third occasion. This pattern was generally found more efficient than patterns I and II.

6. Summary

For sampling upto three occasions, using a two-stage sampling design three different sampling patterns have been examined with partial matching among psu's and ssu's. The relative efficiencies of the estimates obtained for second and the third occasion with different sampling patterns have been studied. It was observed that for sampling on two occasions, partial matching of units at both the stages [sampling pattern (*iii*)] is generally superior to matching among ssu's only [sampling pattern (*ii*)] and its relative efficiency increases with increase in correlation between psu's. Further, partial matching at both stages was found to be as good as matching among psu's only [sampling pattern (*i*)], the relative efficiency gradually decreases as correlation between psu's increases. For sampling on three occasions, in order to estimate the mean on the current occasion, it is better to have partial matching with the immediately preceeding occasion only, *i.e.*, sampling pattern (*iii*) is superior to the other two sampling patterns.

TABLE 1

Relative efficiency of the estimate $\bar{y}_w$ w.r.t. the estimate $\bar{y}_2$ for different values of ρ_b, ρ_w, ϕ, q and s for m=4

ϕ	ρ_b	q=0·5										q=0·75									
		s=0·5					s=0·75					s=0·5					s=0·75				
		ρ_w=0·0	0·5	0·7	0·9	1·0	0·0	0·5	0·7	0·9	1·0	0·0	0·5	0·7	0·9	1·0	0·0	0·5	0·7	0·9	1·0
	0·0	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00
	0·5	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00
0·1	0·7	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00
	0·9	1·00	1·00	1·00	1·00	1·00	1·00	0·99	0·99	1·00	1·00	1·00	0·99	0·99	0·99	1·00	1·00	1·00	0·99	0·99	1·00
	1·0	1·00	0·99	0·99	1·00	1·00	1·00	0·99	0·99	0·99	1·00	1·00	0·99	0·98	0·99	0·99	1·00	0·98	0·97	0·98	0·99
	0·0	1·00	1·00	1·01	1·02	1·03	1·00	1·00	1·01	1·02	1·04	1·00	1·00	1·00	1·01	1·01	1·00	1·00	1·00	1·01	1·02
	0·5	1·00	0·99	1·00	1·01	1·02	1·00	0·99	0·99	1·00	1·03	1·00	0·99	0·99	1·00	1·00	1·00	0·99	0·98	0·99	1·00
1·0	0·7	1·00	0·99	0·99	1·00	1·02	1·00	0·98	0·98	0·99	1·02	1·01	0·98	0·98	0·99	1·00	1·00	0·97	0·97	0·97	1·00
	0·9	1·00	0·98	0·98	0·99	1·01	1·00	0·96	0·96	0·97	1·01	1·00	0·97	0·96	0·96	0·97	1·00	0·95	0·93	0·93	0·97
	1·0	1·00	0·97	0·97	0·98	1·00	1·00	0·95	0·94	0·96	1·00	1·00	0·95	0·93	0·92	0·94	1·00	0·92	0·89	0·88	0·92
	0·0	1·00	1·00	1·01	1·04	1·08	1·00	0·99	0·99	1·03	1·13	1·00	0·99	0·99	1·00	1·02	1·00	0·99	0·98	0·98	1·03
	0·5	1·00	0·99	0·99	1·02	1·07	1·00	0·97	0·96	0·99	1·10	1·00	0·98	0·97	0·96	0·98	1·00	0·97	0·94	0·93	0·98
10	0·7	1·00	0·98	0·98	1·01	1·06	1·00	0·96	0·94	0·97	1·09	1·00	0·97	0·96	0·94	0·95	1·00	0·96	0·92	0·90	0·95
	0·9	1·00	0·97	0·97	0·99	1·05	1·00	0·95	0·92	0·95	1·08	1·00	0·97	0·94	0·91	0·90	1·00	0·95	0·90	0·86	0·90
	1·0	1·00	0·97	0·96	0·97	1·04	1·00	0·94	0·92	0·94	1·07	1·00	0·96	0·93	0·88	0·87	1·00	0·94	0·89	0·83	0·86

TABLE 2

Relative efficiency of the estimate $\bar{y}_w$ w.r.t. the estimate E_2 for different values of ρ_b, ρ_w, ϕ, q and s for m=4

ϕ		$q=0{\cdot}5$										$q=0{\cdot}75$									
		$s=0{\cdot}5$					$s=0{\cdot}75$					$s=0{\cdot}5$					$s=0{\cdot}75$				
	ρ_b	$\rho_w=0{\cdot}0$	0·5	0·7	0·9	1·0	0·0	0·5	0·7	0·9	1·0	0·0	0·5	0·7	0·9	1·0	0·0	0·5	0·7	0·9	1·00
	0·0	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	0·99	0·99	1·00	1·00	1·00	0·99	0·99
	0·5	1·07	1·07	1·07	1·07	1·07	1·07	1·07	1·07	1·07	1·07	1·05	1·06	1·06	1·05	1·05	1·05	1·05	1·05	1·05	1·05
0·1	0·7	1·15	1.16	1·16	1·16	1·16	1·15	1·15	1·15	1·16	1·16	1·13	1·14	1·14	1·14	1·14	1·13	1.14	1·14	1·14	1·14
	0·9	1·31	1·32	1·32	1·33	1·33	1·31	1·32	1·32	1·33	1·33	1·34	1·36	1·36	1·37	1·37	1·34	1·35	1·35	1·36	1·37
	1·0	1·45	1·47	1·47	1·48	1·49	1·45	1·46	1·46	1·48	1·49	1·62	1·66	1·68	1·70	1·72	1·62	1·64	1·65	1·68	1·71
	0·0	1·00	0·99	0·99	0·98	0·97	1·00	0.99	0·98	0·98	0·98	1·00	0·99	0·98	0·96	0·93	1·00	0.99	0·98	0·96	0·94
	0·5	1·04	1·05	1·05	1·06	1·06	1·04	1·04	1·04	1·05	1·06	1·03	1·04	1·04	1·02	1·00	1·03	1·03	1·03	1·01	1·00
1·0	0·7	1·09	1·11	1·12	1·13	1·14	1·09	1·10	1·10	1·12	1·14	1·08	1·09	1·10	1·10	1·08	1·08	1·08	1·08	1·08	1·08
	0·9	1·17	1·21	1·23	1·26	1·29	1·17	1·19	1·20	1·24	1·29	1·16	1·20	1·22	1·26	1·28	1·16	1·18	1·19	1·22	1·27
	1·0	1·24	1·28	1·32	1·36	1·40	1·24	1·26	1·28	1·33	1·40	1·23	1·30	1·35	1·49	1·50	1.23	1·26	1·29	1·36	1·48
	0·0	1·00	0·98	0·97	0·96	0·97	1·00	0·97	0·95	0·95	1·00	1·00	0·98	0·95	0·89	0·81	1·00	0·97	0·94	0·87	0·82
	0·5	1·00	1·01	1·01	1·02	1·05	1·00	0·99	0·97	0·99	1·08	1·00	1·00	0·98	0·94	0·88	1·00	0·98	0·95	0·90	0.89
10	0·7	1·01	1·02	1·02	1·05	1·10	1·01	1·00	0·99	1·01	1·12	1·01	1·01	1·00	0·97	0·93	1·01	0·99	0·96	0·92	0·94
	0·9	1·02	1·03	1·05	1·09	1·15	1·02	1·00	1.09	1·04	1·19	1·01	1·02	1·02	1·01	1·00	1·01	1·00	0·98	0·95	1·00
	1·0	1·02	1·04	1·06	1·11	1·19	1·02	1·01	1·01	1·05	1·22	1·02	1·03	1·03	1·03	1·05	1·02	1·00	0·98	0·97	1·04

TABLE 3

Relative efficiency of the estimate $\bar{y}_3^{(2)}$ w.r.t. the estimate $\bar{y}_3^{(1)}$ for different values of ρ_b, ρ_w, ϕ, q and s for m=4.

ϕ		$q=0.5$										$q=0.75$									
		$s=0.5$					$s=0.75$					$s=0.5$					$s=0.75$				
	ρ_b	$\rho_w=0.0$	0.5	0.7	0.9	1.0	0.0	0.5	0.7	0.9	1.0	0.0	0.5	0.7	0.9	1.0	0.0	0.5	0.7	0.9	1.0
0.1	0.0	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.5	0.99	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	0.99
	0.7	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	0.9	0.99	1.00	1.00	0.99	1.00	0.99	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.01	1.00	1.00
	1.0	1.00	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.0	0.0	1.00	0.99	1.00	1.00	0.98	1.00	0.99	1.01	1.01	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	0.99
	0.5	0.98	0.99	0.98	0.98	0.99	0.99	1.00	1.00	1.01	0.98	0.98	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00
	0.7	0.96	0.99	0.98	0.97	0.98	0.98	1.00	1.01	1.00	0.98	0.97	0.99	0.99	0.99	0.99	0.98	1.01	1.01	1.01	1.00
	0.9	0.96	0.98	0.98	0.97	0.98	0.98	1.01	1.01	1.00	0.98	0.96	0.98	0.99	0.99	0.99	0.98	1.01	1.01	1.00	0.99
	1.0	0.96	0.98	0.99	0.98	0.98	0.98	1.01	1.01	1.00	0.98	0.97	0.99	0.99	0.98	0.99	0.98	1.01	1.00	1.00	0.99
10.0	0.0	0.97	0.97	0.95	0.92	0.92	0.99	1.00	0.99	0.97	0.91	0.97	0.98	0.97	0.96	0.96	0.99	1.00	1.01	1.00	0.95
	0.5	0.93	0.96	0.95	0.91	0.92	0.98	1.01	1.02	0.99	0.91	0.94	0.97	0.97	0.95	0.96	0.98	1.01	1.03	1.02	0.95
	0.7	0.93	0.95	0.95	0.91	0.92	0.97	1.02	1.02	1.00	0.91	0.94	0.97	0.96	0.96	0.96	0.98	1.02	1.04	1.02	0.95
	0.9	0.92	0.95	0.95	0.91	0.92	0.97	1.02	1.03	1.00	0.91	0.94	0.96	0.96	0.95	0.95	0.98	1.02	1.04	1.03	0.95
	1.0	0.92	0.96	0.95	0.91	0.92	0.97	1.02	1.04	1.00	0.90	0.93	0.97	0.96	0.95	0.96	0.97	1.03	1.04	1.02	0.94

TABLE 4

Relative efficiency of the estimate $\bar{y}_3^{(3)}$ w.r.t. the estimate $\bar{y}_3^{(1)}$ for different values of ρ_b, ρ_w, Φ, q and s for m=4

Φ	ρ_b	$q=0\cdot5$										$q=0\cdot75$									
		$s=0\cdot5$					$s=0\cdot75$					$s=0\cdot5$					$s=0\cdot75$				
		$\rho_w=0\cdot0$	$0\cdot5$	$0\cdot7$	$0\cdot9$	$1\cdot0$	$0\cdot0$	$0\cdot5$	$0\cdot7$	$0\cdot9$	$1\cdot0$	$0\cdot0$	$0\cdot5$	$0\cdot7$	$0\cdot9$	$1\cdot0$	$0\cdot0$	$0\cdot5$	$0\cdot7$	$0\cdot9$	$1\cdot0$
	0·0	1·00	1·00	1·00	1·00	0·99	1·00	1·00	1·00	1·00	0·99	1·00	1·00	1·00	1·00	1·01	1·00	1·00	1·00	1·00	1·01
	0·5	1·01	1·01	1·01	1·02	1·02	1·01	1·01	1·01	1·01	1·02	1·01	1·01	1·02	1·01	1·01	1·01	1·01	1·01	1·02	1·01
0·1	0·7	1·05	1·06	1·06	1·06	1·07	1·06	1·06	1·06	1·06	1·07	1·04	1·04	1·04	1·04	1·04	1·04	1·04	1·04	1·05	1·04
	0·9	1·18	1·19	1·20	1·20	1·20	1·18	1·19	1·19	1·20	1·20	1·11	1·11	1·12	1·12	1·13	1·12	1·12	1·12	1·12	1·13
	1·0	1·32	1·34	1·35	1·34	1·35	1·32	1·34	1·34	1·34	1·35	1·19	1·20	1·20	1·21	1·22	1·19	1·20	1·20	1·21	1·22
	0·0	1·00	1·00	1·00	0·99	0·98	1·00	0·99	1·00	0·99	0·98	1·00	1·01	1·01	1·02	1·03	1·00	1·01	1·01	1·02	1·03
	0·5	0·98	0·99	1·00	1·01	1·01	0·99	1·00	1·00	1·00	1·00	0·98	1·00	1·01	1·03	1·04	0·99	1·00	1·01	1·04	1·05
1.0	0·7	0·99	1·03	1·03	1·05	1·05	1·01	1·02	1·03	1·05	1·05	0·99	1·02	1·03	1·06	1·06	1·00	1·02	1·03	1·06	1·08
	0·9	1·05	1·09	1·11	1·15	1·17	1·07	1·08	1·11	1·13	1·16	1·03	1·06	1·09	1·13	1·15	1·05	1·07	1·08	1·12	1·15
	1·0	1·11	1·15	1·20	1·25	1·28	1·13	1·14	1·17	1·22	1·27	1·08	1·11	1·14	1·18	1·21	1·09	1·11	1·12	1·17	1·21
	0·0	0·97	0·98	0·97	0·98	0·95	0·99	0·99	0·99	0·99	0·92	0·97	1·00	1·03	1·08	1·11	0·99	1·01	1·04	1·10	1·15
	0·5	0·93	0·96	0·97	0·99	0·99	0·97	0·98	0·99	0·99	0·95	0·95	0·98	1·03	1·09	1·13	0·98	1·00	1·04	1·11	1·14
10·0	0·7	0·93	0·96	0·97	1·00	1·01	0·97	0·99	0·99	1·00	0·97	0·94	0·98	1·02	1·09	1·15	0·98	1·01	1·04	1·10	1·15
	0·9	0·93	0·96	0·98	1 02	1·06	0·98	0·99	0·99	1·01	1.01	0·94	0·98	1·03	1·11	1·17	0·98	1·01	1·04	1·11	1·19
	1·0	0·92	0·96	0·99	1·08	1·09	0·97	0·98	1·00	1·02	1·03	0·94	0·99	1·03	1·12	1·20	0·98	1·01	1·04	1·12	1·20

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